

TWO-DIMENSIONAL POTENTIAL FLOW IN
A GRAVITATIONAL FIELD WITH A KNOWN
FREE SURFACE

BY

KIN NEE TONG

B.S., Chiao-Tung University, 1945

M.M.E., Syracuse University, 1949

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN THEORETICAL AND APPLIED MECHANICS
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TWO-DIMENSIONAL POTENTIAL FLOW IN A GRAVITATIONAL FIELD WITH A KNOWN FREE SURFACE

The problem of potential flow with free boundaries, which includes flow in open channels, free jets and flow around blunt obstacles, belongs to that class of non-linear problems which have not been solved in a general way. Ever since Helmholtz, Kirchhoff and others started the investigation some seventy years ago, this problem has always been within the attention of mathematicians and engineers. The main difficulty of the problem lies in the fact that the free boundary of the flow is unknown beforehand and the boundary condition to be satisfied on this part of boundary is the non-linear Bernoulli equation.

The lack of a complete knowledge of the boundary renders the mathematical formulation of the problem difficult. It is becoming increasingly evident that in many problems the analytical character of the solutions depends on the assumption made in advance concerning the make-up of the domain of flow. For instance, in the problem of the wake behind a blunt obstacle, the wake may be infinite or finite, and thus a change in the assumption causes a change in the connectivity of the domain as well as the pressure to be assumed on the free boundary. So far analytical work in this field has been limited to the study of free jets and wakes where the gravity can be neglected or does not come into play. Exception is made in the case of the gravity wave problem which, however, is considered as a problem in the small since the motion is only slightly different from the state of rest. Genuine problems in the large with the gravitational force accounted for have been scarcely touched upon.

In view of the fact that most of the difficulty arises because the free boundary is unknown, it is deemed profitable to investigate the problem by assuming a free boundary for the flow and to fit it with an appropriate fixed boundary. Mathematically, the task is far less imposing; practically it has much value in designing a system fit to order.

The method used can be summarized as follows. First, assume a suitable shape for the free boundary, then determine the values of the complex potential on the free boundary by the Bernoulli equation and continue this determination analytically to other parts of the domain. Theoretically, the process of analytical continuation can be carried out without much difficulty by the use of Taylor series expansions. In practice, however, a more expedient means is needed.

In this paper it is assumed that the flow has a connected free boundary describable by a known real analytic function. A real parameter is then chosen for the free boundary profile in such a manner that the velocity potential determined from the Bernoulli equation is also an analytic function of this parameter. Since the velocity potential is equal to the complex potential on the free boundary, the same function will serve for the complex potential in other parts of the flow by allowing the parameter to be complex. The desired analytical continuation is thereby obtained.

Three examples are worked out. The first one has a rectilinear free boundary, the second has a hyperbolic free boundary and the last one has a free boundary profile described by a hyperbolic tangent function. In the first problem, both the complex potential and the velocities are obtained in closed forms. In the second, the velocities are obtained in closed forms. In the last one, a numerical procedure is used, which modifies somewhat the procedure outlined above.

VITA

Kin Nee Tong was born on February 22, 1922, in Shanghai, China. In September, 1940, he entered the National Chiao-Tung University of the same city. Two years later, he moved with part of that university to Chungking, China, due to the outbreak of the war in the Pacific. He served in the Chinese Army in 1944 and 1945. Subsequently he received the degree of Bachelor of Science in June, 1945, from Chiao-Tung University. In February, 1947, he entered the Graduate School of Syracuse University and held a half-time graduate assistantship for two semesters. From February, 1948, to June, 1949, he was an instructor in the Department of Mechanical Engineering of that institution. After receiving the degree of Master of Mechanical Engineering in June, 1949, from Syracuse University, he entered the Graduate College of the University of Illinois. He held a half-time instructorship in the academic years 1949-50 and 1950-51, in the Engineering College of the University of Illinois.



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